

SELF-OSCILLATIONS, PARAMETRIC MODULATION, AND NONLINEAR PARAMETRIC
RESONANCE IN BULK CRYSTALLIZATION PROCESSES

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We consider several nonlinear effects, generated during stability breakdown of the formation process of discrete elements of a new phase (crystals, gas bubbles) in a metastable fluid.

In metastable media, superheated fluids, and supersaturated solvents one often observes in phase transition processes regular periodic variations of temperature, pressure, two-phase system concentrations, and the mean size of heterogeneous inclusions.

In many cases the principal formation mechanism of formation of self-oscillations is smoothed as follows. In a sufficiently fast system-transition into a metastable state a metastability level is reached in it, for which the nucleus formation process is sharply activated, and has a cascade-formation character. The latter is determined by the strongly nonlinear increase in formation rate of elements of the new phase with an increase in superheating, supersaturation, or supercooling. In this case the increase in new phase elements leads to a reduction in the extent of metastability (for example, to a decrease in superheating due to heat extraction by growing bubbles). Following removal of heterogeneous inclusions from the bulk due to their floating or precipitation at the reservoir walls the extent of metastability in the combined elements of the new phase of the medium again increases under the effect of external sources. The cycle described is further repeated. Similar self-oscillating regimes can be observed in processes of different nature, such as heating of a polydisperse fuel, as well as simultaneous flow of polymerization and crystallization when their thermal effects are commensurate. The applied value of investigating periodic heat- and mass-exchange processes with phase and chemical transformations is determined by the necessity of developing effective methods of controlling industrial crystallizers, thermal energy and cryogenic systems.

Attention was paid in [1-4] to stability analysis of the equilibrium regimes of bulk crystal formation and the formation characteristics of the self-oscillating regimes. In the present study we consider the effect of parametric modulations on the characteristics of neutral stability and the nonlinear resonance effects related to them, formed by several control processes with nonstationary regimes by means of parametric oscillations.

For definiteness we consider below crystallization from supersaturated solutions. The analysis provided can be extended without difficulty to crystallization of supercooled alloys or solutions. The solution parameters with a low relative content of crystals is assumed to be uniform within the limits of the considered sufficiently large volume, which corresponds to the hypothesis of ideal mixing of suspensions. To describe nucleus formation we use the results of stationary nucleation theory. The process is assumed to be isothermic. The mass balance equation of the crystallizing material and the kinetic equation for the distribution function of crystals over their radii r are written in the form:

$$\frac{dc}{dt} = Q \left(\frac{c - c_0}{c_0} \right) - \rho \int_{r_*}^{\infty} \frac{d}{dt} \left(\frac{4}{3} \pi r^3 \right) f(t, r) dr, \quad (1)$$

$$\frac{df}{dt} + \frac{\partial}{\partial r} \left(\frac{dr}{dt} f \right) + \gamma(r) f = 0, \quad \frac{dr}{dt} f \Big|_{r=r_*} = J \left(\frac{c - c_0}{c_0} \right). \quad (2)$$

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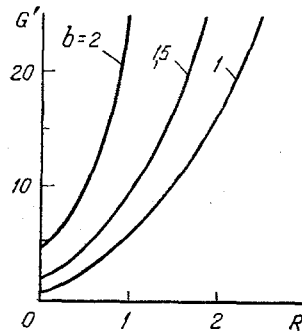


Fig. 1. Neutral stability surface trace in the plane of parameters G' , R as a function of the parameter b for $k = 1$.

The boundary condition in (2) corresponds to equality of the effective flow of crystals of minimum size to the frequency of nucleus formation, which is assumed to be a known function of relative supersaturation and can be determined by the Frenkel-Zel'dovich theory

$$J = C \exp \left[- \frac{A}{\ln^2 \frac{c}{c_0}} \right]$$

or by the phenomenologic Mayer kinetic

$$J = C(c - c_0)^m.$$

In the kinetic equation (2) we neglect random fluctuations in the crystal growth rate dr/dt . Their account would lead to occurrence of diffusion along the r axis and corresponding terms in the equations and boundary conditions (2). The corresponding analysis was carried out in [4]. The parameter γ characterizes the rate of crystal removal from the system.

The crystal growth rate is determined in the general case by an expression of the following type:

$$\frac{dr}{dt} = \frac{1}{\Phi(r)} \beta \left(\frac{c - c_0}{c_0} \right), \quad \Phi = r^{b-1}, \quad \beta = \beta_0 \left(\frac{c - c_0}{c_0} \right)^k. \quad (3)$$

It is convenient to introduce new variables

$$u = \frac{c - c_0}{c_0}, \quad s = \gamma_0 \int_0^r \frac{\Phi(r) dr}{\beta_s}, \quad \tau = \gamma_0 \int_0^t \frac{\beta}{\beta_s} dt, \quad \Gamma = \gamma/\gamma_0, \quad (4)$$

where γ_0 are several characteristic values of the function γ , and u_s is the relative supersaturation corresponding to the stationary process.

Neglecting the effect of initial conditions on the crystal distribution density during the evolution of the asymptotic phase of the process, we obtain in the variables (4)

$$f(t, r) = F(\tau, s) = \Phi(s) \frac{J[u(\tau - s)]}{\beta[u(\tau - s)]} \exp \left\{ -\beta_s \int_0^s \frac{\Gamma(s-z) dz}{\beta[u(\tau - z)]} \right\}. \quad (5)$$

Equation (5) was obtained in [3]. It is noted that, in principle, the parameter γ , characterizing the time of finding crystals in the system, may depend on the distribution function. Here we provide an expression for the crystal distribution function $f(t, r)$ for the case $\gamma = \gamma(\tau) f^n$:

$$f(t, r) = F(\tau, s) = \Phi(s) \left[\frac{\beta^n [u(\tau - s)]}{J^n [u(\tau - s)]} + \frac{n\beta_s}{\Gamma_0} \int_0^s \frac{\Phi^n(s-z) \gamma(s-z) dz}{\beta [u(\tau - z)]} \right]^{-\frac{1}{n}},$$

$$n \neq 0, \quad \Gamma_0 = \gamma_0^{3/(4n+3)} J_s^{4n/(4n+3)}.$$

The last equation makes it possible to generalize all results obtained below for $n = 0$ to situations in which the dependence of λ on the crystal distribution is substantial.

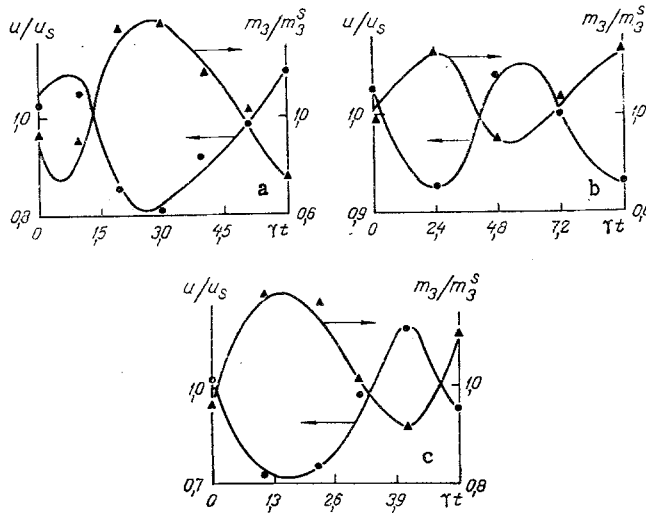


Fig. 2. Comparison of theory and experiments [8] on characteristics of self-oscillations of the relative supersaturation u/u_s and of the third moment of the distribution function m_3/m_3^s for growth of sodium chloride crystals for $b = 1$, $k = 1$, $J = Cu^m$; a) $R = 0.21$; $G'/G'_0 - 1 = 0.28$; b) $R = 0.32$; $G'/G'_0 - 1 = 0.026$; c) $R = 1.6$; $G'/G'_0 - 1 = 0.89$.

Transforming the balance equation (1) by means of (5), we obtain a functional integro-differential equation, determining the evolution of relative supersaturation in the bulk of the crystallizer:

$$\frac{\beta(u)}{\beta_s} \frac{d\xi}{d\tau} - \frac{RQ(u)}{Q(u_s)} + \frac{R\beta(u)}{\Omega} \int_0^\infty \frac{J[u(\tau-s)]}{J_s\beta[u(\tau-s)]} \exp\left\{-\beta_s \int_0^s \frac{\Gamma(s-z) dz}{\beta[u(\tau-z)]}\right\} \frac{r^2(s) ds}{\Phi(s)} = 0. \quad (6)$$

The following new variables and parameters were introduced here:

$$\xi = \frac{u - u_s}{u_s}, \quad R = \frac{Q(u_s)}{c_0 \gamma_0 u_s}, \quad \Omega = \int_0^\infty \exp\left\{-\int_0^s \Gamma(z) dz\right\} \frac{r^2(s) ds}{\Phi(s)}.$$

Based on the stationary analog of Eq. (6), it is easy to describe the stationary crystallization regimes by means of time-independent supersaturation. In this case the crystal distribution function over sizes acquires the form

$$f(r) = F(s) = \Phi(s) \frac{J_s}{\beta_s} \exp\left\{-\int_0^s \Gamma(z) dz\right\}, \quad (7)$$

and the stationary supersaturation is determined by the relation

$$Q(u_s) = \frac{4\pi\rho\beta_s^2}{\gamma_0} \int_0^\infty \frac{F(s) r^2(s) ds}{\Phi^2(s)}. \quad (8)$$

The stationary regime (7), (8) is quite often unstable with respect to small supersaturation fluctuations [2]. To analyze the stability of regime (7), (8) we assume perturbations $|\xi| \ll 1$ and, restricting ourselves to a linear transformation in ξ in (6), we obtain the equation

$$\begin{aligned} \frac{d\xi}{d\tau} + R\xi(B - D) + \frac{G' - RB}{\Omega} \int_0^\infty \xi(\tau - s) \exp\left\{-\int_0^s \Gamma(z) dz\right\} \frac{r^2(s) ds}{\Phi(s)} + \\ + \frac{RB}{\Omega} \int_0^\infty \left[\int_0^s \Gamma(s - z) \xi(\tau - z) dz \right] \exp\left\{-\int_0^s \Gamma(z) dz\right\} \frac{r^2(s) ds}{\Phi(s)} = 0, \end{aligned} \quad (9)$$

in which we introduce the parameters:

$$D = u_s \left. \frac{d \ln Q}{du} \right|_{u=u_s}, \quad B = u_s \left. \frac{d \ln \beta}{du} \right|_{u=u_s}, \quad G' = R u_s \left. \frac{d \ln J}{du} \right|_{u=u_s}. \quad (10)$$

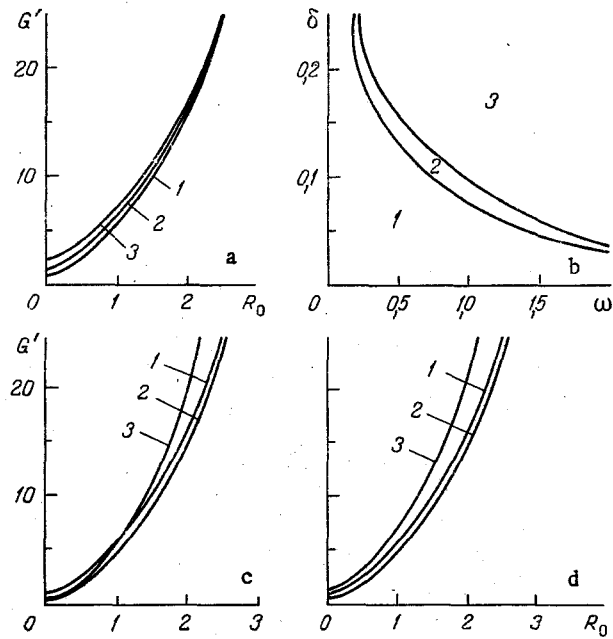


Fig. 3. Effect of parametric modulation on the conditions of violation of stability of stationary regimes for $b = 1$, $k = 1$: a) displacement of neutral stability curves during modulation of the parameter R : 1) $p = 0$, 2) $p = 0.2$, 3) $p = 0.5$; b) region of variation of the amplitude δ and frequency ω of modulating the parameter γ/γ_0 , leading to destabilization (1), stabilization (3), not affecting the stability of stationary regimes (2); c) displacement of neutral stability curves during modulation of the parameter γ/γ_0 : 1) $\delta = 0$, 2) $\delta = 0.04$, 3) $\delta = 0.1$; d) displacement of neutral stability curves during simultaneous modulation of the parameters R and γ/γ_0 : 1) $p, \delta = 0$; 2) $p = 0.05, \delta = 0.04$; 3) $p = 0.3, \delta = 0.1$.

Substituting into (9) a perturbation ξ in the form

$$\xi = \xi_0 e^{\lambda \tau}, \quad (11)$$

we obtain a complex equation for the parameter λ , not derived here due to its awkwardness. The validity of representing ξ in the form (11) to Eq. (9) requires some justification. The matter is that the object of the classical linear stability theory is an ordinary differential equation, which Eq. (9) is not. However, this equation can be reduced to an infinite autonomous system of linear differential equations for ξ and moment distribution functions of crystals over sizes [5]. For this system the representation (11) is fully justified. This equation for ξ makes it possible to write down relations for the neutral stability curve in the space of parameters G', R, B, D . For illustration, Fig. 1 shows the neutral stability curves $G' = S(R, b)$ for the case

$$\Phi = r^{b-1}, \beta = \beta_0 u, \gamma = \text{const}, Q = \text{const}. \quad (12)$$

The instability is determined by the condition $G' > S(R, b)$. Self-oscillating crystallization regimes are generated in the instability region. For small supercriticality $G'/G'_* - 1$ these oscillations can be investigated by small parameter methods, similarly to the way it was done in [2-4]. The essence of this method consists of representing the relative supersaturation in the form of a Fourier series:

$$u = u_s \left[1 + \sum_{n=-2}^2 \Phi_n \exp in\omega\tau \right], \quad \Phi_{-n} = \Phi_n^*, \quad (13)$$

$$\Phi_2 \sim \Phi_0 \sim \Phi_1^2 = \Phi_1 \Phi_{-1} = q.$$

Substituting expansion (13) into Eq. (7), in which the nonlinear terms must be expanded in a Taylor series up to third order of smallness in powers of ξ , and analyzing the relations obtained, one can determine the amplitude q , the frequency ω , and $\Phi_0 = \langle \xi \rangle$. This method is

valid only for small supercriticality. However, the real system parameters often acquire values, corresponding to close approach in the instability region, where the theory of small-amplitude self-oscillations is not valid. This creates the necessity of numerical solution of the nonlinear equation (6), which can be realized by the Aitken–Steffensen iteration method [6, 7]. As a result of the numerical analysis, characteristics of self-oscillations are obtained as a function of supercriticality. Self-oscillations of supersaturation are generated by periodic changes in the most important integral characteristics of the process – the mean crystal radii, their bulk concentration, and the crystal output. This was fixed experimentally in many studies (see, for example, [8–11]). Figure 2 shows the results of comparing the calculated theoretical characteristics with experiments [8], for both small and large supercriticalities. It is easily seen that the mean saturation time decreases with increasing supercriticality, while the crystal mass yield increases. This increase is quite substantial, and for a subcriticality $(G' - G'_0)/G'_0 \sim 1$ it can reach 8–15%, in agreement with the experimental results of [8].

Along with the problems noted above of instability and self-oscillations an important problem is that of stabilization of unstable crystallization regimes. A promising means of its solution is the use of parametric modulation. To investigate the effect of kinetics-modulation of crystal removal from the system and modulation of the removal rate of solution and crystals on the system behavior for mass removal kinetics, crystal growth, and their removal, described by relations (12), we put

$$R = R_0[1 + p\varphi(\tau)], \quad \gamma = \gamma_0[1 + \delta\varphi(\tau)]. \quad (14)$$

Here $\varphi(\tau)$ is a periodic function of time τ . The determination of stability boundaries of the stationary crystallization regime reduces in this case to the search of periodic solutions of the linear equation (9) with coefficients (14), determining the evolution of perturbations of stationary supersaturation u_S :

$$\begin{aligned} \frac{d\xi}{d\tau} + R_0[1 + p\varphi(\tau)]\xi + \frac{R_0(G-1)[1 + p\varphi(\tau)]}{\Omega(\tau)} \int_0^\infty \xi(\tau-s) \exp\left\{-s - \delta \int_0^s \varphi(\tau-z) dz\right\} s^{\frac{3}{b}-1} ds + \\ + \frac{R_0[1 + p\varphi(\tau)]}{\Omega(\tau)} \int_0^\infty \left[\int_0^s [1 + \delta\varphi(\tau-z)] \xi(\tau-z) dz \right] \exp\left\{-s - \delta \int_0^s \varphi(\tau-z) dz\right\} s^{\frac{3}{b}-1} ds = 0, \end{aligned} \quad (15)$$

where $\Gamma(x)$ is the Euler Gamma-function, G is the Jacobi number $J'u_S/J_S$, and

$$\Omega(\tau) = \int_0^\infty \exp\left\{-s - \delta \int_0^s \varphi(\tau-z) dz\right\} s^{\frac{3}{b}-1} ds.$$

To obtain qualitative information on the effect of modulation on stability, the analysis of Eq. (15) is conveniently started with the study of simplest dependences of $\psi(\tau)$ [12]. In particular, we use here a rectilinear stepwise modulation law, for which

$$\varphi(\tau) = \begin{cases} 1, & \frac{2\pi n}{\omega} < \tau < \frac{\pi(2n+1)}{\omega}, \\ -1, & \frac{\pi(2n-1)}{\omega} < \tau < \frac{2\pi n}{\omega}, \end{cases} \quad (16)$$

where n is an integer. From the point of view of technological applications the modulation law (16) is quite simple and easily realizable. The general solution of Eq. (15) on the interval $[-\pi/\omega, \pi/\omega]$ is written in the form:

$$\begin{aligned} \xi_1 = e^{-\alpha_1\tau} (C_1 \sin \beta_1\tau + C_2 \cos \beta_1\tau), \quad -\frac{\pi}{\omega} < \tau < 0, \\ \xi_2 = e^{-\alpha_2\tau} (C_3 \sin \beta_2\tau + C_4 \cos \beta_2\tau), \quad 0 < \tau < \frac{\pi}{\omega}. \end{aligned} \quad (17)$$

The continuity conditions of the solutions (17) at the points $-\pi/\omega, 0, \pi/\omega$ have the form:

$$\begin{aligned} \xi_1(0) = \xi_2(0), \quad \dot{\xi}_1(0) = \dot{\xi}_2(0), \\ \xi_2\left(\frac{\pi}{\omega}\right) = \mu \xi_1\left(-\frac{\pi}{\omega}\right), \quad \dot{\xi}_2\left(\frac{\pi}{\omega}\right) = \mu \dot{\xi}_1\left(-\frac{\pi}{\omega}\right), \end{aligned} \quad (18)$$

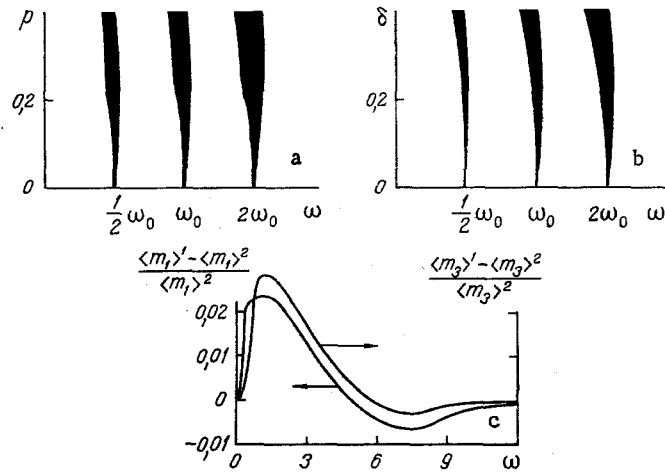


Fig. 4. Effect of parametric modulation on self-oscillating crystallization regimes for $b = 1$, $k = 1$, $J = \text{Cu}^{\text{m}}$: a, b) regions of harmonic, sub- and ultraharmonic capture of natural frequency during modulation of the parameters R and γ/γ_0 , $\omega_0 = 0.92$; c) time-mean variation of the mass output $(m_3)^1$ and of the crystal mean size $(m_1)^1$ at forced oscillations, compared to the mean mass output $(m_3)^2$ and mean crystal size $(m_1)^2$ in the self-oscillating regime with $R_0 = 1$, $p = 0.2$, $\delta = 0$, $\omega_0 = 0.92$, $G' = 6.65$.

where μ is a real parameter, and the dot denotes differentiation with respect to time. The existence condition of a nontrivial solution of system (18) is equating to zero the corresponding characteristic determinant, leading to the equation

$$\cos \frac{\beta_1 \pi}{\omega} \cos \frac{\beta_2 \pi}{\omega} \frac{(\alpha_1 - \alpha_2)^2 + \beta_1^2 + \beta_2^2}{2\beta_1 \beta_2} \sin \frac{\beta_1 \pi}{\omega} \sin \frac{\beta_2 \pi}{\omega} = \mu \operatorname{ch} \frac{(\alpha_1 + \alpha_2) \pi}{\omega}. \quad (19)$$

The periodic solutions, corresponding to the new surface of neutral stability, are determined by the condition $|\mu| = 1$. Besides, in substituting solutions of form (17) into Eq. (15) a number of additional conditions is generated, relating the parameters α_1 , α_2 , β_1 , β_2 . The system of nonlinear algebraic equations obtained was analyzed numerically along with (19). Figure 3 shows the results of applying parametric modulation. The displacement of neutral stability curves during modulation of the parameter R without modulation of γ is illustrated in Fig. 3a; this modulation is the stabilizing factor for any parameters p and ω . For modulation of the kinetics of crystal removal there exist three regions of variation of the amplitude δ and the frequency ω with a qualitatively distinct effect on the stability of the stationary regime (Fig. 3b). In the given case parametric modulation can lead to both stabilization and destabilization of the stationary regime. Besides, the system can seem generally unstable to modulation. The corresponding shift of neutral stability curves is shown in Fig. 3c.

Simultaneous modulation of R and γ is not a result of their simple superposition; this is explained by the fact that these parameters appear in Eq. (16) nonlinearly. Thus, for example, if p is quite large, and δ corresponds to an interval leading to partial destabilization, the total modulation can lead to stabilization on the whole interval of R_0 values. If p and δ are quite small, the basic contribution to variation of the qualitative stability patterns is provided by modulation of kinetics of crystal removal (Fig. 3d). We note that the results obtained here change only quantitatively during transition to a different form of periodic action. In particular, studies of harmonic modulation $\psi(\tau) = \frac{\pi}{2} \sin \omega \tau$ are subject to

all the conclusions formulated above.

The presence of substantial nonlinearities in the system under consideration renders the use of parametric oscillations in the new instability regions quite effective as a control process. Since analytic methods of harmonic balance are quite well developed only for nonlinear equations of the Van der Pol type [13], the analysis of parametric resonance as applied to Eq. (6) was realized numerically. An effect of forcing of frequency of eigenoscillations of external frequency was observed. For quite large modulation amplitude the

eigenfrequency is synchronized with the modulation frequency under the condition that these frequencies are close to each other. The forcing of oscillations also occurs when the eigenfrequency of self-oscillations is close to a frequency, which is larger or smaller by an integer times than the modulation frequency:

$$\omega^{(i)} \approx n\omega^{(e)}, n = \dots, \frac{1}{3}, \frac{1}{2}, 1, 2, 3, \dots$$

In this case the eigenfrequency of forcing oscillations, being respectively larger or smaller than an integer times the modulation frequency, is the so-called sub- or ultraharmonic forcing [14, 15]. In Fig. 4a, b we show the existence regions of these types of oscillations for both types of modulation in the plane in which the coordinates are the modulation amplitude and frequency. The results obtained also indicate that the effects considered are somewhat enhanced with increasing parameter β from 1 to 2 and with increasing index k in a power dependence of β on supersaturation.

Parametric modulation affects the integral characteristics of the process; more precisely, there exists a frequency interval of external action near the natural frequency, in which there occurs an enhancement of the time-mean mass output of crystals $\langle m_3(\tau) \rangle$ by several percents in comparison with the mass output obtained during realization of the process in the self-oscillating regime (Fig. 4c). The mean crystal size $\langle m_1(\tau) \rangle$ is also enhanced in this case.

Thus, parametric modulation can be used both to suppress undesirable instabilities, and to create control of the crystallization process.

NOTATION

A, C, and m, constants determining the nucleation kinetics; b, a parameter introduced in (3); c and c_0 , concentration and thermodynamically equilibrium concentration; C_1 - C_4 , parameters introduced in (17); D and B, parameters introduced in (10); f, size distribution function of crystals; F, function introduced in (5); J and J', nucleation rate and its derivative; G, Gibbs number; G', a parameter introduced in (10); G'_0 , the G' value on neutral stability curve; k, a parameter introduced in (3); m_k , moment of the crystal distribution function over sizes of order k; r and r_* , radius of the crystal and the critical nucleus radius; R_0 , a parameter introduced in (14); R and p, parameters introduced in (10) and (14); q, squared amplitude of the fundamental harmonic of the supersaturation disturbance; Q, mass flow of the substance dissolved into the system; s, a variable introduced in (4); t, time; u, dimensionless supersaturation; u_S , stationary u value; α_1 , α_2 , β_1 , β_2 , parameters introduced in (17); β , β_0 , a function and a parameter introduced in (3); β_S , stationary value of the function β ; γ , γ_0 , kinetics of crystal removal from the system and its characteristic value; δ , a parameter introduced in (14); λ , parameter introduced in (11); μ , a parameter introduced in (18); ξ , supersaturation disturbance; ξ_0 , parameter introduced in (11); ρ , crystal density; τ , dimensionless time; φ , function introduced in (14); Φ , a function introduced in (3); Φ_n , amplitude of the n-th harmonic of the supersaturation disturbance; ω , frequency; ω_0 , frequency value at the neutral stability surface; Ω , a parameter introduced in (6); the asterisk denotes the complex conjugate; the angular brackets denote averaging over time; the subscript (i) refers to the natural frequency of self-oscillations; and (e) to the parametric modulation frequency.

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NONSTATIONARY PROCESSES IN OPTICAL FIBER FORMATION.

1. STABILITY OF THE DRAWING PROCESS

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The stability of quartz glass melt flow in the deformation domain is investigated in the linear hydrodynamic stability approximation as a function of the velocity coefficient, the drawing rate, and the temperature modes of optical fiber formation.

One of the fundamental quality indices of an optical fiber is the constancy of its geometric dimensions along the length. Fluctuations of the lightguide diameter cause nonstationary processes during its formation, for instance, fluctuations of the melt viscosity in the deformation zone, small fluctuations of the feeding and drawing velocities, inhomogeneity of the ingot, etc. Since different perturbations are inevitably present in any real process of optical fiber fabrication, the sensitivity of the lightguide dimensions to small fluctuations in the drawing parameters near their stationary values is of great interest. Closely related to the problem of investigating the reaction of the optical fiber drawing process to external perturbing effects is the problem of its stability because it governs the domain of the parameters where continuous fiber formation is possible.

Instability of the process can be caused by two mechanisms: cohesion breakaway of the lightguide (the tensile stress exceeds its rupture strength), and hydrodynamic instability (small perturbations increase without limit in time and cause fracture of the liquid jet or a periodic change in the thickness of the fiber being formed appears, i.e., so-called drawing resonance is observed). The first fracture mechanism during the drawing of quartz lightguides is associated with underheating of the quartz glass melt, i.e., with too high a value of the viscous friction and is not examined here.

Investigation of the stability of the fiber formation process was performed first for the case of drawing from a filler of a continuous polymeric or vitreous textile fiber in isothermal [1, 2] and nonisothermal conditions [3-5]. However, the application of these results directly to production of optical fiber is difficult (the isothermal model is too rough a generalization and mainly fiber drawing of polymers was examined in [3-5] and the energy equation being used cannot adequately describe the heat transfer process in lightguide production). The stability of optical fiber drawing was studied directly in [6-9] but the stability was investigated in [6-8] only within the framework of the hydrodynamic model. The ingot heating conditions were not examined here while the temperature distribution in the equations of motion was taken into account parametrically by giving the viscosity by a function of the longitudinal coordinate. The system of governing equations in [9] is actually borrowed completely from [3, 5], therefore, the remark formulated above relative to [3, 5] also refers to [9]. Let us note that the dependence of the drawing stability on the temperature conditions for fiber formation is shown convincingly in [8, 9]. It follows from the survey presented that the drawing stability problem for optical lightguides is insufficiently investigated. For a correct solution of the problem posed the heat exchange process during fiber production must be examined more completely since it is apparently governing.

1. Stability of the optical fiber drawing process is investigated in this paper within the framework of the linear theory of stability on the basis of a quasi-one-dimensional model [10]. Fiber formation is considered under simple uniaxial tension of a Newtonian fluid with

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